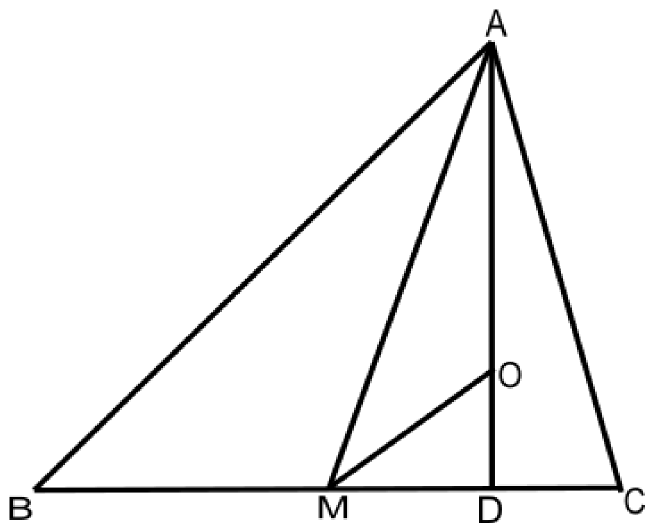

In the picture, in ΔABC, AD is altitude, AM is median and O is the Orthocentre.

Prove:

$2(AM^2 + OM^2) - OA^2 = 4R^2$, **where R is the circumradius of ΔABC.**

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Author's Solution

Construction :

Draw the circumcircle of ΔABC.

Mark its circumcentre at E. Draw diameter AEF. Join FO. Let FO& BC intersect at N. Produce AD to cut the circumcircle at K. Join BF, CO, CK & FK.

Proof:

$\angle AKF = 90^\circ \quad (\because AF \text{ is diameter})$

$\Rightarrow$ FK$\|$ BC

$\Rightarrow$ BFKC is an isosceles trapezium

$\Rightarrow \angle FBN = \angle KCN$ -------------- (1)

And BF=CK -----------------------(2)

CD is the perpendicular bisector of OK

(As per Orthocentre theorem, page 22 of the book "Rajaclimax's theorems on Geometry")

$\Rightarrow \angle OCN = \angle KCN$ -------------(3)

And CO = CK -----------------------(4)

(1), (2) ,(3) & (4) $\longrightarrow$

BF = OC

$\angle FBN = \angle NCO$

$\angle BNF = \angle CNO$ (vertically opposite)

$\Rightarrow \Delta NBF \cong \Delta NCO$

$\Rightarrow$BN = NC ------------------------(5)

And FN = NO -----------------------(6)

(5) $\rightarrow$ N & M are same points (M is the midpoint of BC - given)

$\therefore$ Now, let us call N as M. (see figure-2)

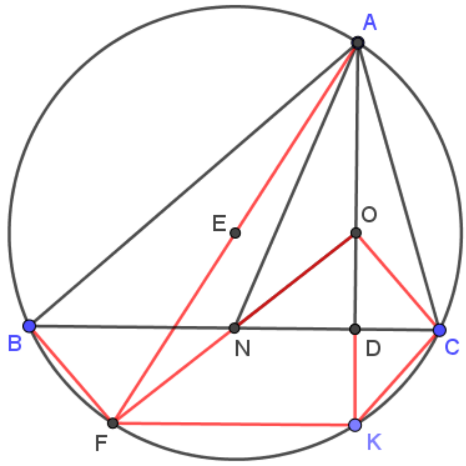

Figure -1

(6) $\rightarrow$ AM is the median of ΔAFO

$\therefore$ As per Apollonius Theorem,

$$AF^2 + AO^2 = 2(AM^2 + OM^2)$$

$$\Rightarrow 4R^2 = 2(AM^2 + OM^2) - OA^2$$ -------------Proved

*****

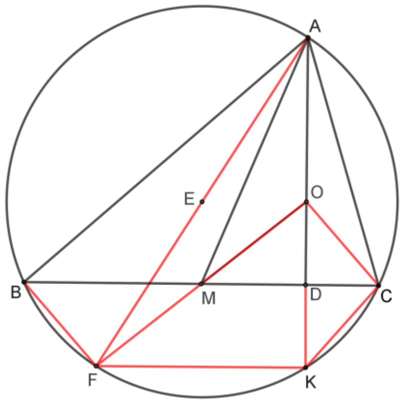

Figure -2

Solution given by
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